WITTGENSTEIN ON LOGIC, TRUTH, AND REALITY: A RESPONSE TO MADDY'S *THE LOGICAL MUST*

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ABSTRACT

PHILIP BOLD: Wittgenstein on Logic, Truth, and Reality:
A Response to Maddy's *The Logical Must*
(Under the direction of Alan Nelson)

One might have thought that mathematical and logical propositions are true, or correspond to reality, in just the same way as empirical propositions. I challenge this natural analogy from the perspective of later Wittgenstein. By responding to Penelope Maddy's recent interpretation of Wittgenstein's remarks on logic, I argue that Wittgenstein was interested in examining this analogy to undermine the temptation to inquire into, investigate, or explore a distinctively logical reality, or aspect thereof. I apply Wittgenstein's therapeutic methods to Maddy's inquiry into the ground of logical truth in order to show that Maddy's interpretation is mistaken, as well as to illuminate Wittgenstein's perspective on logic and mathematics.

To Alan, Kyle and Krasi

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LIST OF ABBREVIATIONS

BB	*The Blue and Brown Books*
BT	*The Big Typescript*
LFM	*Lectures on the Foundations of Mathematics*
PG	*Philosophical Grammar*
PI	*Philosophical Investigations*
RFM	*Remarks on the Foundations of Mathematics*
TLP	*Tractatus Logico-Philosophicus*

The correct explanation of the propositions of logic must assign to them a unique status among all propositions.

Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, 6.112

We feel that mathematics stands on a pedestal—this pedestal it has because of a particular role that its propositions play in our language games.

Ludwig Wittgenstein, *Remarks on the Foundations of Mathematics*, VII, §6

I. INTRODUCTION

On the face of it, the following three claims are on a par.

(1) The diameter of the earth is roughly 7,917.5 mi.

(2) There are exactly 3 primes between 2 and 10.

(3) If the stove is smoking, then the chimney must be broken.

That is, (1) – (3) all immediately strike us as *assertions*. Each, in one way or another, tells us something about the world. (1) makes an assertion about the diameter of the earth, (2) makes an assertion about the number of primes between 2 and 10, (3) makes a claim about what must be going on with the chimney if the stove is smoking. We might say, (1) is an *empirical claim*, corresponding to geographical facts, (2) is a *mathematical claim*, corresponding to facts about numbers, and (3) is a *logical claim*, corresponding to the logical connections between facts about the world, or worldly events.[1] (1) – (3), then, may be said to have different contents, but they are all fundamentally similar in the following way: They each correspond to some aspect of reality, either empirical, mathematical, or logical. In other words, though (2) and (3) might differ from (1) in terms of their contents, they correspond to reality in much the same way that empirical claims do.

[1] One might think that (3), like (1), can be understood simply as an empirical claim. I agree that it can. If this bothers you, replace (3) with the more cumbersome "If 'the stove is smoking', and 'whenever the stove is smoking, the chimney must be broken', then 'it *must* be the case that the chimney is broken'." I present (3) as a bare conditional for the sake of simplicity and readability.

In this paper, I challenge the natural picture articulated above from the standpoint of later Wittgenstein.[2] One might have thought that logic and mathematics correspond to reality just as empirical propositions correspond to reality. That is, that (2) and (3) correspond to reality analogously to (1). Wittgenstein was well aware of a natural tendency, especially amongst philosophers, to form such an analogy between the truth of empirical propositions and the truth of logical or mathematical propositions. The analogy recommends metaphysical theorizing about the distinctive reality, or aspect thereof,[3] to which logic and mathematics, in some sense, correspond: Just as we can discover, explore, and inquire into the nature of empirical reality, we can discover, explore, and inquire into the nature of mathematical and logical reality. Wittgenstein thought that once this analogy is properly scrutinized, and the distinctive roles of empirical and logical or mathematical propositions are appreciated, the temptation to investigate logical or mathematical reality is diminished.

I will argue for these claims by responding to Penelope Maddy's recent interpretation of Wittgenstein on logical truth.[4] According to Maddy, Wittgenstein's remarks on logic are compatible with her view that rudimentary logic represents the contingent logical structure of the world. Maddy argues that, insofar as Wittgenstein departs from this view, his departure can be fully explained by an unprincipled rejection of scientific method. I argue that this interpretation is

[2] In particular, the Wittgenstein of *Philosophical Investigations* and *Remarks on the Foundations of Mathematics*.

[3] By this I mean that the analogy recommends metaphysical theorizing about either (i) the distinctive reality to which mathematics and logic correspond, or (ii) the distinctively logical aspect of the very same reality to which empirical propositions correspond. (i) is more closely associated with "3rd realm" Platonism, whereas (ii) is more closely associated with naturalistic views of mathematics and logic. Not much will ride on this distinction in my paper: I only note this to say that Wittgenstein was not simply reacting to Platonistic tendencies. Any metaphysical theorizing about mathematical or logical reality that relies on the analogy is within the purview of his critique.

[4] Maddy 2014.

incorrect. Wittgenstein's therapeutic strategies, when carried out correctly, render suspect the motivations for Maddy's inquiry into the ground of logical truth. As Maddy is explicit that one of the motivations for her philosophical inquiry into the ground of logical truth is that logic and mathematics say something about the world in much the same way that science does, her misinterpretation of Wittgenstein will serve as a helpful illustration. That is, we can understand Wittgenstein's rejection of the analogy between empirical and mathematical or logical propositions, along with its philosophical significance, by drawing out the implications for Maddy's own view. In doing so, we'll thus see an important way in which her interpretation of Wittgenstein is mistaken.

In Section 1, I present Maddy's naturalistic, or "Second Philosophical", view of logic, along with her interpretation of later Wittgenstein. In Section 2, I introduce the Therapeutic Philosopher in order to get a better grip on how Wittgenstein would have reacted to the Second Philosopher's project. Finally, in Sections 3 and 4, I argue that the *Remarks on the Foundations of Mathematics* contains a therapeutic strategy that is designed to undermine the analogy that motivates Maddy's inquiry: the analogy between empirical and logical or mathematical propositions. In Section 3, I begin by connecting Wittgenstein's remarks on 'truth', very generally, to the Second Philosopher's inquiry into *logical* truth. In Section 4, I argue that Wittgenstein was actively working to dislodge the analogy between empirical and logical or mathematical truth.

II. SECOND PHILOSOPHY, LOGICAL STRUCTURE, AND LATER WITTGENSTEIN

In a number of places,[5] Penelope Maddy develops a naturalistic answer to the question "what is the ground of logical truth?". She offers

> a roughly empirical account of logical truth derived from Kant: logic is grounded in the structure of our contingent world; our basic cognitive machinery is tuned by evolutionary pressures to detect that structure where it occurs (Maddy 2014, ix).

Maddy calls this structure, shared by mind and world, "Kant-Frege Structure", or "KF Structure" for short, as it is taken to be a successor of Kant's view that human cognition is constituted by pure categories of thought, which are in turn governed by logical principles and subject-predicate structure. She accepts Kant's view that, when understood empirically, logic is true of objective reality. However, she departs from Kant's transcendental perspective, according to which logic is not true of the world itself, but is the contribution of thought. Logic is thus ideal, according to Kant, and the necessity and a priority of logic is explained by the pure categories, which are requirements for human cognition. Maddy proposes that we collapse Kant's transcendental inquiry into his empirical inquiry, and claims that rudimentary logic simply reveals the objective logical structure of the world. Further, the account is modified so as to include appropriate Fregean updating: subject-predicate structure is replaced with objects in relations, and syllogistic inference is replaced with rudimentary logic, as we know it today, so as to include universal and existential

[5] Maddy (2002, 2007, & 2014).

quantification. According to Maddy, it's true that if 'it's either red or it's green' and 'it's not red' then, 'it *must* be green', because this inference accurately represents the objective KF-structure of the world. The *must* of logic thus depends crucially on the structure of the world, as discovered by empirical science: 'it *must* be green', given the truth of the preceding claims, because the structure of the world, in some sense, guarantees that this is so.

Maddy's view is articulated and defended within a particular methodological perspective that she calls "Second Philosophy". Second Philosophy, as a successor to the methodological naturalism of Quine, aims to answer traditional philosophical questions using methods resembling those of ordinary science. Second Philosophy is not to be characterized as a particular doctrine, or set thereof; instead, it can only be explained by describing the distinctive manner in which the Second Philosopher approaches traditional philosophical questions.

> [M]y Second Philosopher investigates the world beginning from her ordinary perceptual beliefs, gradually developing more sophisticated observational techniques and correctives, eventually ascending to theory formation and confirmation, all in the sorts of empirical ways usually labeled 'scientific'. (Maddy 2014, 2).

Second Philosophical methodology is not simply "to answer philosophical questions: look to what 'science' tells us!". Such a method would presuppose an answer to the demarcation problem, and would ignore the fact that the Second Philosopher appeals directly to scientific data, rather than what scientists have to say about it, in order to develop her own theories.[6] The Second Philosopher, in other words, does not simply develop her theories according to the philosophical pronouncements of scientists, instead, she makes use of their data, and comes to her own

[6] "We philosophers, speaking of [The Second Philosopher] in the third person, will say that such an inquirer operates 'within science', that she uses 'the methods of science', but she herself has no need of such talk. When asked why she believes that water is H_2O, she cites information about its behavior under electrolysis and so on; she doesn't say, 'because science says so and I believe what science says'" (Maddy 2007, 15).

philosophical conclusions. Further, the Second Philosopher considers alternative techniques and strategies along with their merits as she proceeds. That is, insofar as they seem promising, the Second Philosopher is open to applying methods that are not typically labeled 'scientific'.

Maddy's Second Philosophical answer to the question "what is the ground of logical truth?" is stated in terms of three major theses. First, "rudimentary logic is true of the world insofar as it is a KF-world, which in many but not all respects it is" (Maddy 2014, 28). The Second Philosophical support for this thesis comes from our ordinary perceptual evidence that the world of trees, planets, cats, and people "does come (roughly) divided into these individual objects" (Maddy 2014, 20). The caveat here—"insofar as the world is a KF-world"—is important: The Second Philosopher is persuaded that macro- and micro-physical reality exemplify different logical structures. For instance, various findings in quantum mechanics lead to important questions as to whether quantum phenomena conform to rudimentary logic.

> [I]f logic does depend on the sort of KF-structuring that's not clearly present in the micro-world, then we should expect breakdowns in rudimentary logic there, and this is just what we do find. (Maddy 2014, 27)

Notwithstanding the findings of microphysics, there is overwhelming evidence that the macro-physical world we encounter in our most ordinary dealings possesses KF-structure.

Second, our basic cognitive machinery detects and represents the KF-structure of the macro-physical world. Maddy here draws from data in cognitive science.

> Current work supports the notion that prelinguistic infants, given ordinary maturation (as opposed to learning), perceive a world of cohesive, solid bodies that travel on continuous paths and preserve their identity through time and when out of sight. … Very young children, again by ordinary maturation, are also capable of classifying objects by their features, individual and relational, and of recognizing both Boolean-style correlations between these properties … and dependencies between one situation and another. (Maddy 2014, 29).

Maddy concludes from the psychological data, "our ability to detect the presence of the world's KF-structures is part of our most fundamental cognitive endowment" (Maddy 2014, 29).

Third, and finally, "human cognitive machinery is as it is because we live in a largely KF-world and interact almost exclusively with its KF-aspects" (Maddy 2014, 29). In other words, it's not a mere coincidence that we accurately represent KF-structure. The Second Philosopher, again, inspired by the data gathered by developmental psychologists, concludes that human cognition is the product of evolutionary pressures. Our cognition has KF-structure because cognizing the world in this manner has led to successful engagement with macroscopic environments, and our success is to be explained by the fact that the world indeed has KF-structure. Maddy's view is thus

> a brand of realism—logic reflects objective truths about the world—but without many of the features that typically accompany such realism: logical truth isn't necessary, but contingent on the presence of requisite structures; logic doesn't describe a world of abstracta, but our own familiar physical world. (Maddy 2014, 30)

Thanks to evolutionary pressures, we successfully detect and represent the objective KF-structure of the macrophysical world. That is, KF-structure was in the world, waiting to be discovered, before humans had anything to say about the matter.

In her recent book, *The Logical Must: Wittgenstein on Logic*, Maddy compares her distinctive account of logical truth with the views of several historical figures: Kant (as we've seen), early Wittgenstein, and later Wittgenstein. The aim of the book is to simultaneously contribute to the understanding of these historical views, as well as to further clarify her Second Philosophical view of logic.[7] The basic strategy that runs through the text is to show what sort of

[7] "My hope is that this investigation will prove its worth in two ways, one primary, one secondary. The primary aim is simply historical—to understand Wittgenstein better—but the method is unusual: the idea is to use the Second Philosopher's view of logic, and its roots in Kant, to illuminate Wittgenstein's view on logic, both early and late. If successful, this approach, this atypical perspective, should serve to focus attention in unexpected ways and draw out less familiar features. Along the way, the process of compare

modification would be required to transform each of these philosophers into genuine Second Philosophers of logic. In the case of Kant, as we've seen, collapsing the transcendental into the empirical will do the job, along with a bit of Fregean updating.[8] In the case of early Wittgenstein, one must reject what she takes to be an underlying thesis in the *Tractatus*, viz., "the priority of sense", according to which "the sense of our representations must be given prior to, independently of, any facts about how the contingent world happens to be" (Maddy 2014, ix).[9] In the case of later Wittgenstein, one must deny Wittgenstein's apparent legislation against scientific methodology.[10] Throughout the rest of the paper, I will focus on this last component of Maddy's project, viz., her interpretation of later Wittgenstein.

Maddy's interpretation of Wittgenstein's views on logic draws primarily from his remarks on rule following. According to Maddy, Wittgenstein sees logic as one among many instances of rule following, and thus sees it as vulnerable to the same sorts of confusions that he addresses in the *Philosophical Investigations*. On Maddy's interpretation, Wittgenstein addresses confusions about logic and rule following by appealing to ordinary observations about the dependence of our practices on contingent features of the world. In particular, Maddy takes Wittgenstein to insist that our interests, our natural inclinations, and very general features of our macroscopic environments are sufficient to ground our logical practices.

> we humans share a range of interests and motivations; we find the same traits salient, the same applications natural; we react to similar training in similar ways. In addition, the viability of our rule-following practices also depends on various obvious and general facts about the world. … These ordinary facts about the world … are enough to ground our

and contrast should also reveal new aspects of the Second Philosopher's position—and this is the secondary aim" (Maddy 2014, 3).

[8] Maddy 2014, Chapter 2.

[9] Maddy 2014, Chapter 4.

[10] Maddy 2014, Chapter 7.

> practice of continuing series, including the correctness of 1002 and the incorrectness of 1004 in continuing '+2' after 1000. (Maddy 2014, 68-69)

Maddy thus agrees with commentators who

> soon largely agreed that Wittgenstein himself doesn't espouse the 'paradox' of §201 in his own voice, that he means what he says when he begins the second paragraph of that section with the observation that 'there is a misunderstanding here' (Maddy 2014, 66).

Any attempt to seek some further metaphysical grounding for these practices, e.g., that logic conveys necessary features of the world, the understanding of which completely determines correct and incorrect application, is misguided. The trio of our interests, our natural inclinations, and very general features of the world is sufficient. First, as a matter of practical interest, our inferential practices allow us to engage in many important tasks, whether we're calculating the amount of time it will take us to walk to the bus, or the cost of some bit of lumber, or whatever. Second, our inclination to accept inferences like "it's either red or green; it's not red; therefore, it *must* be green" are explained by certain shared natural tendencies. That is, we've accepted this practice *in part* because we, for the most part, find inferences like this to be natural and unquestionable. And third, general features of the world, for instance, that objects don't suddenly pop in and out of existence, they don't suddenly grow and shrink, etc., further sustain our inferences. Our practices, in other words, depend on, and are made useful by, widespread and predictable regularities.

> These practices…wouldn't work if we humans didn't share various interests and practices, didn't find the same features salient, the same connections natural; they wouldn't work if the world itself were too irregular to support them. But we do, and it is, so the ordinary training succeeds. (Maddy 2014, 71)

Were the world different in any of these respects, alternative practices would likely be implemented. But, as things stand, our current practices are sufficiently grounded.

Although logic is not the immediate focus of the rule following considerations contained in the *Investigations*, Maddy notes that similar conclusions can be drawn from Wittgenstein's claims about logic in the *Remarks on the Foundations of Mathematics*.

> 'But doesn't e.g. 'f(a)' have to follow from '(x)f(x)' if '(x)f(x)' is meant in the way we mean it?'—And how does *the way* we mean it come out? Doesn't it come out in the constant practice of its use? and perhaps further in certain *gestures*—and similar things. (RFM I, §10)

> It is important that in our language—our natural language—'all' is a fundamental concept and 'all but one' less fundamental; i.e. there is not a *single* word for it, nor yet a characteristic gesture. (RFM I, §15)

Maddy claims that we can draw many of the same lessons from these passages.

> This sounds like just another example—'all', alongside '+2', 'rose', and 'game'—of how our meanings rest on the consistency of our practices, on our shared reactions to training, on what we find natural. (Maddy 2014, 75)

However, Maddy thinks there is an additional challenge to consider. It's not enough, according to Maddy, to say that our practices of logical inference are grounded in our natural inclinations and constant use of logical terms. There is the further question of why these logical practices are better than genuine alternatives. For instance,

> Our 'or' and 'not' underwrite the truth-preserving inference from 'it's red or green' and 'it's not red' to 'it's green', but 'tonk' licenses the truth-squandering move from 'it's red' to 'it's red tonk it's green', and from there to 'it's green'. We can fix the meaning of 'tonk' in the same general ways as we manage to fix the meaning of '+2'—in fact, we have—but using 'tonk' in place of 'or' and 'not' would lead us astray. (Maddy 2014, 76).

The question then is why 'or' and 'not' deserve a privileged status over 'tonk'. In what sense would 'tonk' lead us astray? Maddy notes that Wittgenstein raises a similar concern:

> What would happen if we made a different inference—*how* should we get into conflict with the truth? (RFM I, §5)

As in the rule following considerations of the *Investigations*, Maddy gathers that Wittgenstein would have answered this question with a number of ordinary observations. First, using 'tonk' would lead us to conflict with the truth because 'tonk' permits us to infer falsehoods from truths. For instance, from 'Donald Trump is president', one can infer 'Donald Trump is president tonk the moon is made of cheese', and from this infer 'the moon is made of cheese'. And so, clearly, using 'tonk' rather than 'or' and 'not' leads us from truth to falsehood. Second, Maddy claims,

> The practice of inferring is also based on our interests and goals: we want to know about the world around us; we enhance that knowledge by having a reliable method of moving from truths to truths. … Using 'tonk' defeats the purpose of inferring. *Both* correct application of existing rules *and* the choice between possible rules rests on the familiar Wittgensteinian trio of our interests, our nature, and the world's regularities. (Maddy 2014, 78)

Thus, our use of rudimentary logic is privileged because alternative conventions, such as the use of 'tonk', would require us to infer falsehoods from truths, and would be unreliable given our interests to know and understand the world around us.

Maddy takes herself to be in complete agreement with Wittgenstein on all of these points. That is, our logical practices are grounded in our shared interests, natural inclinations, and very general features of the world. There is no need to appeal to the metaphysical necessity of logic in order to more fully ground logical truth. Maddy, however, sees herself as filling in some of the details, in particular, by adding that our interests include those of knowing about the world, and that the general features on which logical truth depends include objective KF-structure.

> [G]iven our interest in and motivations for describing the world, logic is reliable insofar as the world has this very general KF-structuring, and our natural responses play the role they do because our basic cognitive mechanisms are tuned to detecting those structures; [the Second Philosopher] goes on to spell out the details of these claims and to marshal empirical evidence in support. The two positions [viz., Wittgenstein's and the Second Philosopher's] would be entirely compatible. (Maddy 2014, 101)

Since the world is carved up into relatively stable objects and relations, and permits logical inferences recommended by rudimentary logic, our logical practices are successful and reliable. Maddy sees nothing in Wittgenstein's remarks that would render dubious these additional details.

The problem, however, is that Wittgenstein explicitly banishes theory and explanation from philosophy.

> We may not advance any kind of theory…all *explanation* must disappear. (PI §109)
>
> Philosophy must… in the end only describe [the actual use of language]…It cannot… justify it. (PI §124)
>
> Philosophy… neither explains nor deduces anything. (PI §126)

Further, he explicitly denies the relevance of scientific methods endorsed by the Second Philosopher.

> Our considerations must not be scientific ones. (PI §109)
>
> Our craving for generality has another main source: our preoccupation with the method of science. … Philosophers constantly see the method of science before their eyes, and are irresistibly tempted to ask and answer questions in the way science does. This tendency is the real source of metaphysics, and leads the philosopher into complete darkness. … Philosophy really *is* 'purely descriptive'. (BB, 18)

Wittgenstein is not engaging in any kind of scientific inquiry, nor does he intend to make any special use of the claims or findings of scientists. Treating philosophical problems like scientific questions, according to Wittgenstein, "leads the philosopher into complete darkness". Maddy concludes

> At this point, obviously, Wittgenstein and the Second Philosopher part company. Her scientific inquiries can't be viewed as filling in Wittgenstein's picture, after all, because his is a picture that cannot, for whatever reason, be elaborated empirically. (Maddy 2014, 103)

How then can Maddy claim that her interpretation illuminates the views of later Wittgenstein, given that he so clearly rejects the methods of the Second Philosopher? Maddy here distinguishes two questions: (1) whether Wittgenstein put forward any reasons to reject the Second Philosopher's methods, and (2) whether these reasons are any good (Maddy 2014, 107). According to Maddy, insofar as Wittgenstein offered reasons against the Second Philosopher's approach to logical truth, these reasons are unpersuasive and lack philosophical significance. In other words, everything of immediate philosophical interest can be preserved if we simply reject Wittgenstein's ban against scientific method.

As a matter of biography, Wittgenstein did indeed express aversion to scientific ways of thinking.[11,12] But the Second Philosopher isn't troubled by "Wittgenstein's personal distaste for science" (Maddy 2014, 107). However, Maddy searches for principled grounds for Wittgenstein's rejection, and comes up empty handed.

> What this reading leaves out is Wittgenstein's personal antipathy to science, but this by itself carries no force. Nothing we've seen indicates that Wittgenstein's anti-theoretical stance is organically intertwined with the heart of the insights into rule-following and logical truth that he shares with the Second Philosopher. (Maddy 2014, 122)

Maddy then concludes that the only modification one must make to the later Wittgenstein in order to align him with her brand of naturalism is to remove

[11] "I was talking about in Cambridge and passed a bookshop, and in the window were portraits of Russell, Freud and Einstein. A little further on, in a music shop, I saw portraits of Beethoven, Schubert and Chopin. Comparing these portraits I felt intensely the terrible degeneration of only a hundred years" (quoted in Monk 1999, 299, from Drury's recollections in Rhees 1984, 112).

[12] "It isn't absurd, e.g., to believe that the age of science and technology is the beginning of the end for humanity; that the idea of great progress is a delusion, along with the idea that the truth will ultimately be known; that there is nothing good or desirable about scientific knowledge and that mankind, in seeking it, is falling into a trap. It is by no means obvious that this is not how things are" (quoted in Monk 1990, 485, from Wittgenstein 1980, 56e).

> the prohibition against science. Once that's gone, the road is clear for the elaboration proposed by the Second Philosopher. (Maddy 2014, 122)

Further, Maddy claims that, when this is done, nothing essential to Wittgenstein is lost.

> [W]e could locate nothing of substance in Wittgenstein's rejection of science, of the kind of thing the Second Philosopher pursues to fill in their shared picture of what grounds our logical practices. … [N]othing essential to the late Wittgenstein is lost when the prohibition against science is removed (Maddy 2014, 125).

Given that Wittgenstein offered no philosophically interesting reasons to reject ordinary scientific methods, and the Second Philosopher's contributions are, in all other respects, consistent with Wittgenstein's philosophical remarks, Maddy's view of logic survives Wittgensteinian scrutiny, and salvages everything that is essential to his views.

In the rest of the paper, I will argue that Maddy's assessment here is incorrect. Maddy's Second Philosophical contributions to Wittgenstein's own remarks about logic are incompatible with his views for reasons quite apart from his aversion to the application of science to philosophical questions. We can get at a more fundamental divergence between Wittgenstein and the Second Philosopher by subjecting Maddy's question "what is the ground of logical truth?" to scrutiny. I will argue that, for reasons that can be naturally drawn out of his existing remarks, Wittgenstein would have deemed confused Maddy's question as well as her Second Philosophical answer. In the next section, I'll briefly spell out Wittgenstein's distinctive methodology so that we can more easily see how it would be applied to the Second Philosopher's perplexities.

III. THE SECOND PHILOSOPHER MEETS THE THERAPEUTIC PHILOSOPHER

I will argue that, by Wittgenstein's own lights, the central question of Maddy's inquiry "what is the ground of logical truth?" rests on confusion. In other words, Wittgenstein's divergence from Maddy is not simply to be explained by his contempt for scientism, but, instead, questions of methodological practice aside, the content of her inquiry would have appeared suspect.

In her efforts to find principled Wittgensteinian objections to the Second Philosopher's view of logic, Maddy considers a response along similar lines.

> One approach is to take Wittgenstein as holding that his therapeutic philosophizing is all there is to the subject: there are no genuine philosophical problems; all apparent philosophical problems are generated by misunderstandings and mistaken preconceptions; the only genuine philosophical activity is to treat troubled philosophers, to help them free themselves from these merely apparent perplexities by straightforward description of obvious facts. (Maddy 2014, 107)

However, Maddy claims, none of the confusions Wittgenstein aims to clear away in his therapeutic remarks about logic and rule following apply to the Second Philosopher. According to Maddy, the only confusions addressed in these remarks are the insistence on the priority of sense, found in the *Tractatus*, and the insistence that logic "must be special in some way, not simply one among many cases of rule following" (Maddy 2014, 108). As the Second Philosopher rejects the priority of sense, and agrees with Wittgenstein that logic is not "special" as it compares to other rule following practices, she is not the proper subject of Wittgensteinian therapy. Maddy concludes that the objection on the grounds that philosophy ought to conform to "an austere form of therapy" is

"unlikely a persuasive case against the Second Philosopher's approach to the question of logical truth" (Maddy 2014, 119). The Second Philosopher objects, "the evidence available to her investigation of our inferential practices shouldn't be subjected to an artificial limit" (Maddy 2014, 109). Again, Wittgenstein might have deemed Maddy's central question confused, but, according to Maddy, principled grounds for such a rejection are lacking.

Before addressing what Wittgenstein would have taken to be a central confusion behind Maddy's inquiry, it is important to first get clearer on how Wittgenstein's therapeutic methodology is supposed to work. Here I think it will be useful to follow Maddy's clever approach in her characterization of the Second Philosopher. That is, rather than committing Wittgenstein to any specific doctrine, or set thereof, it will be less misleading to introduce a practitioner of his methods, call her the "Therapeutic Philosopher"; we can get a grip on her methods by carefully observing the distinctive way in which she approaches philosophical questions. Like the Second Philosopher, the Therapeutic Philosopher does not offer a hard and fast answer to questions of demarcation. That is, just as the Second Philosopher does not offer necessary and sufficient conditions for what is to count as 'science', the Therapeutic Philosopher does not offer necessary and sufficient conditions for what is to be deemed an instance of 'philosophical confusion'. On this understanding of the Therapeutic Philosopher, one can read the *Investigations* as a series of examples of how the Therapeutic Philosopher approaches philosophical questions. Just as one learns how to use 'block', 'slab', and '+2' by being offered examples of correct application, one learns Wittgenstein's methods by seeing how he works through particular philosophical questions and confusions.

> The real discovery is the one that enables me to break off philosophizing when I want to.—The one that gives philosophy peace, so that it is no longer tormented by questions which bring *itself* in question.—Instead, a method is now demonstrated by examples, and the

> series of examples can be broken off.—Problems are solved (difficulties eliminated), not a *single* problem. (PI §133)

To answer whether Maddy's question, and her broadly scientific approach to philosophical inquiry, holds up to scrutiny, as she claims they do, we need to see how the Therapeutic Philosopher would engage with her question, her theory, and whatever presuppositions might be lying in the background of her inquiry.

Here, then, is a rough sketch of how the Therapeutic Philosopher approaches philosophical questions. Familiar with philosophical inquiry, and sensitive to certain trends in philosophical thinking, the Therapeutic Philosopher recognizes a philosophical question when she sees one, and wonders whether, like others she has encountered, the one that she now faces rests on some kind of confusion—in our case, the question "what is the ground of logical truth?". To make some headway on this, she submits questions to a distinctive kind of scrutiny: carefully connecting the terms of the inquiry to their use in ordinary linguistic practice, describing those practices in detail, and constructing artificial language games to throw light on our more complex linguistic conventions when they do not immediately admit of a clear or perspicuous overview.[13] Other methods are introduced, and selected, on the basis of trial and error: Does method X clarify issues, or simply replace one source of confusion for another? The approach thus requires creativity and

[13]"A main source of our failure to understand is that we don't have *an overview* of the use of our words.—Our grammar is deficient in surveyability. A surveyable representation produces precisely that kind of understanding which consists in 'seeing connections'. Hence the importance of finding and inventing *intermediate links*" (PI §122).

"Our clear and simple language-games are not preliminary studies for a future regimentation of language—as it were, first approximations, ignoring friction and air resistance. Rather, the language-games stand there as *objects of comparison* which, through similarities and dissimilarities, are meant to throw light on features of our language" (PI §130).

openness to experimentation with new methods. At the end of the day, however, they are selected strictly on the basis of their effectiveness for clarifying issues, and dispelling confusion.[14]

Methods that the Therapeutic Philosopher has found promising in various contexts can be usefully deployed when a new philosophical problem is introduced. For instance, Wittgenstein diagnosed a number of philosophical confusions as crucially relying on misleading analogies, in particular, analogies between different forms of expression.[15, 16] However, whether an analogy is misleading is not itself sharply defined. One discovers whether an analogy was misleading by engaging in therapeutic inquiry, and seeing whether her inclination to talk as she does persists.[17]

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[14] "All *explanation* must disappear, and description alone must take its place. And this description gets its light—that is to say, its purpose—from the philosophical problems. These are, of course, not empirical problems; but they are solved through an insight into the working of our language, and that in such a way that these working are recognized—*despite* an urge to misunderstand them. The problems are solved, not by coming up with new discoveries, but by assembling what we have long been familiar with. Philosophy is a struggle against the bewitchment of our understanding by the resources of our language" (PI §109).

[15] For instance, in a section of *The Big Typescript* titled "Philosophy Points out the Misleading Analogies in the Use of our Language", Wittgenstein writes, "If I rectify a philosophical mistake and say that this is the way it has always been conceived, but this is not the way it is, I must always point out an analogy according to which one had been thinking, but which one did not recognize as an analogy" (BT, 302e).

[16] For a particular case, one can see the method applied in *The Blue and Brown Books* to Augustine's confusions about time: "The contradiction which here seems to arise could be called a conflict between two different usages of a word, in this case the word "measure". Augustine, we might say, thinks of the process of measuring a *length*: say, the distance between two marks on a travelling band which passes us, and of which we can only see a tiny bit (the present) in front of us. Solving this puzzle will consist in comparing what we mean by "measurement" (the grammar of the word "measurement") when applied to a distance on a travelling band with the grammar of that word when applied to time. The problem may seem simple, but its extreme difficulty is due to the fascination which the analogy between two similar structures in our language can exert on us. (It is helpful here to remember that it is sometimes almost impossible for a child to believe that one word can have two meanings.) (BB, 26)

[17] "One of the most important tasks is to express all false thought processes so true to character that the reader says, "Yes, that's exactly the way I meant it". To make a tracing of the physiognomy of every error.

Indeed, we can only prove that someone made a mistake if he (really) acknowledges this expression as the correct expression of his feeling.

For only if he acknowledges it as such, *is* it the correct expression. (Psychoanalysis)

What the other person acknowledges is the analogy I'm presenting to him as the source of his thought" (BT, 303e).

> When we say that by our method we try to counteract the misleading effect of certain analogies, it is important that you should understand that the idea of an analogy being misleading is nothing sharply defined. No sharp boundary can be drawn round the cases in which we should say that a man was misled by an analogy. … It is, in most cases, impossible to show an exact point where an analogy begins to mislead us. … The cases in which particularly we wish to say that someone is misled by a form of expression are those in which we would say: "he wouldn't talk as he does if we were aware of this difference in the grammar of such-and-such words, or if he were aware of this other possibility of expression" and so on. Thus we may say of some philosophizing mathematicians that they are obviously not aware of the difference between the many different usages of the word "proof"; and that they are not clear about the difference between the uses of the word "kind", when they talk of kinds of numbers, kinds of proofs, as though the word "kind" here meant the same. (BB, 28-29)

Thus, one of the distinctive methods of the Therapeutic Philosopher is to examine whether a question or theory relies on a misleading analogy between different forms of expression. She then, by exhaustively working through particular examples, draws out differences between these forms of expression, e.g., numerical expressions, different applications of 'proof', uses of the word 'kind', and so forth, in order to dislodge the analogy that is the source of philosophical perplexity. The criterion for determining whether a misleading analogy is indeed the source of perplexity is whether, after carefully examining the differences between forms of expression, one willingly changes her way of speaking. This is not the only method to be found in Wittgenstein's texts, but it is nonetheless very central, and, as we shall see, it will play an important role in our investigation of Maddy's inquiry.

Maddy is right to claim that, if Wittgenstein were to immediately rule out her question "what is the ground of logical truth?" as inherently confused, merely on the basis of an austere conception of philosophy, whereby *all* philosophical questions are inherently misguided, then this rejection would be arbitrary and unpersuasive. But now that we have a better grip on the distinctive approach of the Therapeutic Philosopher, it should be clearer that this is not the way things should go. The Therapeutic Philosopher is more experimental and open-minded than Maddy has perhaps made her out to be. A philosophical question, for the Therapeutic Philosopher, is treated as the

subject of investigation: to seek out the source of perplexity, and to see whether it survives a distinctive kind of scrutiny, i.e., one whereby we carefully examine our ordinary ways of speaking, and, when necessary, compare our ordinary practices to artificially constructed language games.[18] In the rest of the paper, I argue that the Therapeutic Philosopher can find ample resources for the treatment of the Second Philosopher's inquiry in the very texts that Maddy appeals to in support of her own interpretation of Wittgenstein.

[18] "The philosopher treats a question; like an illness" (PI §255).

IV. WHAT IS LOGICAL 'TRUTH', ANYWAY?

How, then, should the Therapeutic Philosopher approach Maddy's inquiry into the ground of logical truth? For starters, the Therapeutic Philosopher recognizes that this is a philosophical question, and suspects that, like other philosophical questions she has encountered, it is rooted in some kind of confusion. She also notices that Maddy answers it by offering a metaphysical thesis: Rudimentary logic represents the objective, and contingent, logical structure of reality. The Therapeutic Philosopher thus notes the immediate temptation to theorize, and wonders what is the source of such a temptation. The question is not immediately dismissed, but becomes the subject of therapeutic investigation.

What next? One might begin by thinking more carefully about 'logical truth', as it is one of the central terms in the Second Philosopher's question. That is, we can ask, what is 'logical truth'? Better to get clearer on this before we investigate into the grounds for it! In fact, Wittgenstein was keenly aware of problems that can arise from confusions about truth, very generally. Let's take a brief detour into Wittgenstein's remarks about truth, and see whether, from them, we can draw some lessons about the Second Philosopher's inquiry.

In PI §130f, Wittgenstein is concerned with the temptation to draw very general conclusions about the nature of propositions on the basis of their superficial similarities, in particular, that we apply to them the predicates 'true' and 'false'. The fact that we apply 'true' to

a wide variety of propositions can make it appear as if all such propositions are fundamentally similar.

> Now it looks as if the explanation—a proposition is whatever can be true or false—determined what a proposition was, by saying: what fits the concept 'true', or what the concept 'true' fits, is a proposition. So it is as if we had a concept of true and false, which we could use to ascertain what is, and what is not, a proposition. What *engages* with the concept of truth (as with a cog-wheel) is a proposition. (PI §136)

This is related to what Wittgenstein takes to be a confusion contained in the *Tractatus*: a desire to find "the general form of the proposition".[19] However, like the concept of a 'game', the concept of a proposition is found to be a family resemblance concept, not admitting of a rigidly defined essence, but instead can be explained only by enumerating examples.

> But haven't I got a concept of what a proposition is, of what we understand by "proposition"?—Indeed, we do; just as we also have a concept of what we understand by "game". Asked what a proposition is—whether it is another person or ourselves that we have to answer—we'll give examples. … [I]t is in *this* way that we have a concept of a proposition. (PI §135)

However, there is a temptation to think that a theory of the general form of a proposition can be offered, since propositions are what we call 'true' or 'false'. Thus, a substantive theory can be formulated on the basis of a simple observation: Propositions are all fundamentally the same, in that they are bearers of truth and falsity.

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[19] "It now seems possible to give the most general propositional form: that is, to give a description of the propositions of *any* sign-language *whatsoever* in such a way that every possible sense can be expressed by a symbol satisfying the description, and every symbol satisfying the description can express a sense, provided that the meanings of the names are suitably chosen.

It is clear that *only* what is essential to the most general propositional form may be included in its description—for otherwise it would not be the most general form.

The existence of a general propositional form is proved by the fact that there cannot be a proposition whose form could not have been foreseen (i.e. constructed). The general form of a proposition is: This is how things stand" (TLP, 4.5).

> At bottom, giving "This is how things are" as the general form of propositions is the same as giving the explanation: a proposition is whatever can be true or false. For instead of "This is how things are", I could just as well have said "Such-and-such is true". (PI §136)

One thus suspects that the essence of propositions can be discovered by way of 'truth'. But the confusion here is addressed in the remaining bits of the passage: truth and falsity are not independent concepts that are then applied to propositions. Instead, the predicate 'true' belongs to the very same language game that its corresponding proposition belongs to. In order to understand 'true', one must understand the use of the proposition to which it is applied.

> [T]his is a bad picture. It is as if one were to say "The chess king is *the* piece that one puts in check". But this can mean no more than that in our game of chess only the king is put in check. Just as the proposition that only a *proposition* can be true can say no more than that we predicate "true" and "false" only of what we call a proposition. And what a proposition is, is in *one* sense determined by the rules of sentence formation …, and in another sense by the use of the sign in the language-game. And the use of the words "true" and "false" may also be a constituent part of this game; and we treat it as *belonging* to our concept 'proposition', but it doesn't *'fit'* it. (PI §136)

"True" and "false" do not 'fit' our concept of proposition in the way that, say, "red" and "not-red" might fit or fail to fit one's concept of apples-in-my-fridge. We have an independent grip on "red" and "not-red", in that we know "red" applies to fire-engines, blood, The Republican Party, "not-red" does not apply to such things, and so forth. We can then determine whether the apples in my fridge are red by determining whether "red" applies to them, since we have an independent grip on "red". That is, there are known applications of "red" that have nothing to do with the apples in my fridge, relevant sources of comparison, or paradigms, that allow us to say whether "red" applies to the apples in my fridge.[20] In contrast, "true" and "false" are not independent of our concept of

[20] Compare: "Now think of the following use of language: I send someone shopping. I give him a slip of paper marked "five red apples". He takes the slip to the shopkeeper, who opens the drawer marked "apples"; then he looks up the word "red" in a chart and finds a colour sample next to it; then he says the series of elementary number-words—I assume that he knows them by heart—up to the word "five", and for each

propositions; truth is bound up with our concept of proposition, which in part explains why the identification of 'p is true' with, simply, 'p', is so seductive.[21, 22] Just as it is not informative to be told that the chess king is the piece one puts in check,[23] it is likewise uninformative to say that propositions are what we call "true" and "false", since we now need to understand the distinction between 'propositions' and 'non-propositions', which can only be determined by examining the variety of language games to which propositions belong.

One might try to turn things around by claiming that we can get an independent grip on 'true' by recognizing that 'true' only applies to propositions with some property P. For instance, 'true' applies only to propositions that, in some sense, correspond with reality. However, this does not get us any further, at least, not until we have some independent grip on "correspondence with reality". Wittgenstein reportedly addresses such a temptation directly.

> But is a proof just constructing a proposition? Doesn't it show also that the proposition is true? But this isn't satisfactory. To say proposition p is true is just the same as to say p.

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number-word he takes an apple of the same colour as the sample out of the drawer.—It is in this and similar ways that one operates with words" (PI, §1)

[21] Indeed, the correctness of the famous "T-Schema"—'P' is true iff P—is one of the few shared assumptions in philosophy.

[22] "For what does a proposition's '*being true*' mean? 'p' is true = p. (That is the answer.)

So we want to ask something like: under what circumstances do we assert a proposition? Or: how is the assertion of the proposition used in the language-game? And the 'assertion of the proposition' is here contrasted with the utterance of the sentence e.g. as practice in elocution,—or as *part* of another proposition, and so on.

If, then, we ask in this sense: "Under what circumstances is a proposition asserted in Russell's game?" the answer is: at the end of one of his proofs, or as a 'fundamental law' (Pp.). There is no other way in this system of employing asserted propositions in Russell's symbolism" (RFM I, Appendix III, §6).

[23] Not ordinarily, at least. For instance, if someone had learned the rules of chess, but hadn't yet learned that the piece that one puts in check is *called* 'the king'—the rules were explained without naming this piece—, then one would learn something new by being told that the king is the piece that one puts in check, i.e., that this piece is *called* 'the king'. But without knowing what is the piece that one puts in check, being told that *the king* is the piece that one puts in check would be uninformative. One must know the role of the king in order for learning its name to be helpful or informative.

> You might say, "Can't we explain what we mean by 'is true'? For example, to say that p is true means that it corresponds with reality, or that it is in accordance with reality."
>
> Saying this need not be futile at all.—"What is a good photograph?" "One which resembles a man." We explain the words "good photograph" by means of "resemble", etc. This is all right if we know what "resemble" means. But if the technique of comparing the picture with reality hasn't been laid down, if the use of "resembles" isn't clear, then saying this is no use. For there are many different techniques of comparison and many different kinds of resemblance. For instance, one thing may be said to resemble another if it is a projection of it; but there are many different modes of projection—of representing an object. (LFM, 68-69)

A theory of the general form of propositions that appeals to their underlying similarity, viz., that they are all 'true', and that all things that are 'true', in some sense, 'correspond to reality', simply pushes the question back, and in a way that is just as destructive to the assumption that all propositions are fundamentally similar. That is, "correspondence with reality" now requires clarification, which can only be achieved by examining particular examples of its use. But, given that standards of "correspondence" will vary depending on what this concept is applied to—e.g., family portraits, Newton's Second Law, or claims about Watson as stated in a Sherlock Holmes mystery novel—the temptation to find some fundamental, and stable, underlying similarity between all things that we call propositions is unlikely to withstand examination of particular language games to which they belong. These considerations do not answer all confusions associated with 'true' that might remain, but they put pressure on the assumption, inclination, and temptation to think that all propositions are fundamentally similar because they are the things to which we apply 'true' and 'false'.

A relevant comparison here are Wittgenstein's remarks about the temptation to think "Every word in our language signifies something".

> If we say, "Every word in the language signifies something", we have so far said nothing *whatever*; unless we explain exactly *what* distinction we wish to make. (PI §13)
>
> Suppose someone said, "*All* tools serve to modify something. So, a hammer modifies the position of the nail, a saw the shape of a board, and so on."—And what is modified by a

> rule, a glue-pot and nails?—"Our knowledge of a thing's length, the temperature of the glue, and the solidity of a box."—Would anything be gained by this assimilation of expressions?— (PI §14)
>
> Of course, what confuses us is the uniform appearance of words when we hear them in speech, or see them written or in print. For their *use* is not that obvious. Especially when we are doing philosophy! (PI §11)

Just as it is not yet an accomplishment to say that every word in our language signifies something, it is likewise not an accomplishment to say that propositions are the sorts of things that can be "true" or "false". A claim of this sort is uninformative unless we say more about what sort of distinction we are trying to draw, and, on the face of it, the generalization about the relationship between truth and propositions blatantly ignores the diverse array of uses associated with the many things we call propositions. The uniform appearance of 'true' and 'false' which gives rise to this hasty generalization is thus a source of confusion.[24]

So far, I don't think we have much more than a warning from the Therapeutic Philosopher: Do not allow yourself to be misled by the uniform application of "true" and "false" to all of the things we call propositions. Look to the diverse uses of propositions both to understand our concept of 'truth', as well as to understand our concept of 'proposition'.[25] How then is this supposed to

[24] Wittgenstein, as reported by his students, is quite explicit about his intention to treat misunderstanding and confusion by weakening the grip of misleading analogies, that is, by warning against the mere surface similarity of different kinds of expressions: "What kind of misunderstandings am I talking about? They arise from a tendency to assimilate to each other expressions which have very different functions in the language. We use the word "number" in all sorts of different cases, guided by a certain analogy. We try to talk of very different things by means of the same schema. This is partly a matter of economy; and, like primitive peoples, we are much more inclined to say, "All these things, though looking different, are really the same" than we are to say, "All these things, though looking the same, are really different." Hence I will have to stress the differences between things, where ordinarily the similarities are stressed, though, this too can lead to misunderstandings" (LFM, 15).

[25] This is one of many "reminders" that Wittgenstein presents so that we might unveil the sources of philosophical confusion: "The work of philosophers consists in assembling reminders for a particular purpose" (PI §127, 3rd Edition). In this case, a reminder to examine the use of our expressions in order to eliminate philosophical perplexity.

apply to the Second Philosopher's inquiry into the ground of logical truth? These warnings bear relevantly on Maddy's inquiry insofar as she is willing to make a hasty analogy between various things that we call 'true' and 'false', various kinds of proposition. Indeed, Maddy is explicit in her book *Second Philosophy*, that such a move is part of the motivation for her project, namely, an analogy between the truth of empirical propositions and logical propositions.

> Though Frege's understanding of logic as factual in *more or less the same sense as physics* is not widely shared, his resulting opinion that logical laws are normative—and thus that psychologism is inadequate—enjoys a near-universal consensus to this day. (Maddy 2007, 201, emphasis added)

She continues in a footnote:

> As we'll see, the Second Philosopher is one of the few to follow Frege in this, though her logical facts aren't features of a 'third realm' of non-spatiotemporal, causal abstracta—e.g., thoughts and truth functions—as Frege takes them to be. (Maddy 2007, 201, note 7)

Further, Maddy claims:

> My own interest in finding [a Second Philosophical theory of logic] springs partly from the shortcomings of views just rehearsed and partly from an embarrassingly hazy impression from elementary arithmetic: 2 + 2 = 4 seems to me to report something about the world, and that something seems closely connected to logic. (Maddy 2007, 206)

It is clear then that at the heart of the Second Philosopher's inquiry into the ground of logical truth lies an analogy between empirical propositions and logical propositions: They are both true in roughly the same sense, i.e., they both report something about the world. Why should we think that this is central to the Second Philosopher's inquiry? The reason is simple: If logical and mathematical propositions are not in the business of reporting things about the world in the way that, say, propositions of physics are, then it is difficult to see what motivation there might be for the Second Philosopher's claim that logical propositions "represent and detect the objective logical structure of the world" (Maddy 2014, 60. If the functions of logical and empirical propositions are

sufficiently different, and it is false that logical propositions represent the world in roughly the same way that physical propositions do, then there is little motivation for investigation into the alleged structure of the world that they are supposedly in the business of reporting. I argue in the following section that Wittgenstein was actively working to dislodge a natural, but misleading, analogy between empirical and logical propositions, in part to weaken the temptation to inquire into a distinctively logical reality, or aspect thereof.

V. THE UNDERLYING ANALOGY: EMPIRICAL AND LOGICAL PROPOSITIONS

Wittgenstein reportedly comments directly on the temptation to form an analogy between empirical and logical propositions,[26] concerned directly with the tendency to infer on the basis of this facts about mathematical reality.

> Professor Hardy is comparing mathematical propositions to propositions of physics. This comparison is extremely misleading.
>
> "To mathematical propositions there corresponds a reality"—if you take this in the sense of "Some mathematical propositions we affirm", then it is harmless but meaningless. (LFM, 240)

So, there is at least some evidence that Wittgenstein was well aware of the very analogy that compels the Second Philosopher to investigate "the ground of logical truth".[27] We need not rest

[26] Following Maddy, I'm going to use 'mathematics' and 'logic', as well as 'mathematical propositions' and 'logical propositions', more or less interchangeably (Maddy 2014, p. 86). In doing so, I do not mean to encourage ignoring important differences between Wittgenstein's discussions of logic, and his discussions of mathematics. However, as Maddy notes, Wittgenstein was willing to draw a tight connection between mathematics and logic: "'But doesn't it follow with logical necessity that you get two when you add one to one, and three when you add one to two?, and isn't this inexorability the same as that of logical inference?'—Yes! It is the same" (RFM I, 5). As we'll see later, further evidence for this connection is how fluidly Wittgenstein transitions from examples of logic to examples of mathematics. Since the differences between mathematics and logic are not important for our purposes, I am simply going to ignore them here. Logical and mathematical propositions are importantly similar in at least the following sense: Wittgenstein is actively attempting to distance both kinds of proposition from empirical propositions.

[27] Wittgenstein seemed to have been interested in drawing out these differences—that is, between empirical and mathematical propositions—in remarks as early as those collected in *Philosophical Grammar*: "Of course that isn't correct, but if it were correct, nothing in my imaginary description would change. If you call the medical discovery "the discovery of a proof that the man died two hours ago" you must go on to say that this discovery does not change anything in the grammar of the proposition "the man died two hours ago". The discovery is the discovery that a particular hypothesis is true (or: agrees with the facts). We are so accustomed to these ways of thinking, that we take the discovery of a proof in mathematics, sight unseen,

our case on these lecture notes, but, nonetheless, I think they shed light on Wittgenstein's remarks about logic in the RFM. For instance, Wittgenstein says directly that he is pointing to the fundamental differences between mathematical and empirical propositions, and I suggest that he is doing this, in part, in order to address a tempting analogy, one which recommends inquiry into a distinctively logical or mathematical reality, or aspect thereof.

> I should like to say: "When I believe that a x b = c—and I do sometimes have such beliefs—do say that I have them—I am not believing the mathematical proposition, for that comes at the end of a proof; I am believing that this is the formula that comes in such-and-such a place, which I shall obtain in such-and-such a way, and so on".—And this does sound as if I were penetrating the process of believing such a proposition. Whereas I am merely—in an unskillful fashion—pointing to the *fundamental* difference, together with an apparent similarity, between the roles of an arithmetical proposition and an empirical proposition. (RFM I, §110)

An apparent similarity between arithmetical and empirical propositions is that they are both, in some sense, the objects of belief—that is, we can say of both kinds of proposition that we 'believe' them. This might make it appear as if they are similar in some fundamental way, just as the fact that we can apply 'true' to each kind of proposition might make them seem importantly similar. The fundamental difference that Wittgenstein would like to note in this passage is that one cannot understand an equation like a x b = c, and thus cannot be said to believe it, without having some sense of its application, namely, where it stands in a proof or calculation. Although one can be said to 'believe' both empirical and mathematical propositions, Wittgenstein wants to emphasize the

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as being the same or similar. We are wrong to do so because, to put it concisely, the mathematical proof couldn't be described before it is discovered.

The "medical proof" didn't incorporate the hypothesis it proved into any new calculus, so it didn't give it any new sense; a mathematical proof incorporates the mathematical proposition into a new calculus, and alters its position in mathematics. The proposition with its proof doesn't belong to the same category as the proposition without the proof. (Unproved mathematical propositions—signposts for mathematical investigation, stimuli to mathematical constructions.)" (PG, 370-71).

different circumstances in which we would make such assertions, as this reveals their distinctive uses and functions.

> For in certain circumstances I do *say*: "I believe that a x b = c". What do I *mean* by this?—What I *say*!—But what *is* interesting is the question in what circumstances I say this and what is characteristic of them in contrast to those of a statement like: "I believe it is going to rain". For what preoccupies us is this contrast. What we require is a picture of the employment of mathematical propositions and of sentences beginning "I believe that…", where a mathematical proposition is the object of belief. (RFM I, §110)

Why does Wittgenstein want to examine the employment of mathematical propositions? So that we can better understand the differences between empirical and mathematical propositions. Why, then, is he interested in this contrast? What I want to suggest here is the following. The contrast is important to Wittgenstein because one of the sources of metaphysical theorizing about logical and mathematical reality is a misleading analogy between propositions that straightforwardly report something about the world—empirical propositions—and mathematical propositions. As the remark above suggests, we need not, and will not, examine in greater detail what exactly are 'empirical propositions'. For now, we can make do with examples like 'it's going to rain', 'people have an easy time calculating single-digit sums', 'there are Bengal tigers in Nepal', and so forth. Attention to more complicated examples, for instance, having to do with fundamental physical laws, the various cognitive models proposed by contemporary psychologists, or the details of Darwin's evolutionary theory, go beyond what can be accomplished in this paper. I only intend to make the following simple point: Wittgenstein was actively working to dislodge the analogy between empirical and logical propositions, as it is a source of confusion, and temptation to theorize about mathematical or logical reality.[28] This interpretation militates against Maddy's

[28] Compare Wittgenstein's (reported) remark that "Sometimes what is meant by agreement with reality is quite clear. But in a certain number of cases it doesn't determine what we are to do. … [T]here are cases where we don't collate—for example, the tautologies of logic. And of course as the situation gets more and more complicated, God knows what process we should call collation. How do we collate Darwin's theory?

theory of KF-structure, as much as any other metaphysical theory of logical truth and logical reality, and will thus suffice to show that Maddy has not captured everything that is essential to the later Wittgenstein's views of logic.

Further evidence for this interpretation comes from Wittgenstein's insistence that we liken mathematical propositions to 'rules', 'techniques', 'inventions', and 'conventions' that can be evaluated on the basis of their usefulness and practicality, but nothing else.

> The mathematician is an inventor, not a discoverer.[29] (RFM I, §168)
>
> I learned empirically that this came out this time, that it usually does come out; but does the proposition of mathematics say that? I learned empirically that this is the road I traveled. But is *that* the mathematical statement?—What does it say, though? What relation has it to these empirical propositions? The mathematical proposition has the dignity of a rule.
>
> *So* much is true when it's said that mathematics is logic: its moves are from rules of our language to other rules of our language. And this gives it its peculiar solidity, its

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Just look. It is surprising" (LFM, 69). Following Wittgenstein, I won't get caught up in the gritty details of 'empirical propositions', and how *exactly* they can be said to be true, or correspond with reality. As long as we have a firm enough grip on the simple examples, and can draw important differences between the role of such propositions and logical propositions, this should go some way toward dislodging the analogy that is at the heart of the Second Philosopher's inquiry. I will admit, however, that a more scrupulous Therapeutic Philosopher will not ignore these details. My main purpose here is not to carry out the Therapeutic Philosopher's work in excruciating detail, but to give a taste of her methods, and to offer strong evidence that such methods can be found in Wittgenstein's own writings.

[29] This remark is apt to mislead one into thinking Wittgenstein is committed to a thesis, viz., "Mathematics is an invention; there are no mathematical discoveries!". But here, as in many cases where Wittgenstein appears to be expressing a theoretical commitment, it is better to understand him as recommending a picture for the sake of comparison, so that the picture which is the immediate source of confusion—for instance, one according to which mathematicians make discoveries about a distinctively mathematical reality, or aspect thereof—will lose its grip. Here it is especially helpful to compare the RFM to closely related remarks recorded by Wittgenstein's students, and collected in LFM: "One talks of mathematical discoveries. I shall try again and again to show that what is called a mathematical discovery *had much better be called a mathematical invention*. In some of the cases to which I point, you will perhaps be inclined to say, "Yes, they had better be called inventions"; in other cases you may perhaps be inclined to say, "Well, it is difficult to say whether in this case something has been discovered or invented" (LFM, 22, emphasis added).

"It was not a discovery that 125/5 = 25; for this result is merely part of the use of the symbols.—This has to do with what I said, that 'mathematical discoveries' are *better called* inventions. He invented a technique; the reason why the technique is interesting and useful is an extra-mathematical consideration" (LFM, 82, emphasis added).

> unassailable position, set apart. (Mathematics deposited among the standard measures.) (RFM I, §165)
>
> The proof doesn't *explore* the essence of the two figures, but it does express what I am going to count as belonging to the essence of the figures from now on.—I deposit what belongs to the essence among the paradigms of language. The mathematician creates *essences*. (RFM I, §32)
>
> There correspond to our laws of logic very general facts of daily experience. They are the ones that make it possible for us to keep on demonstrating those laws in a very simple way (with ink on paper for example). They are to be compared with the facts that make measurement with a yardstick easy and useful. (RFM I, §118)

Wittgenstein appears in a number of places to insist on the differences between empirical and mathematical propositions, and also to liken mathematical propositions to things that are not typically understood as reporting things about the world, e.g., conventions, inventions, and rules. One might think that the last text above directly supports Maddy's interpretation, according to which, the truth of logical or mathematical propositions importantly depends on the very general KF-structure of the natural world that they detect and represent. But this is to overlook the comparison with yard sticks at the end of the remark: "[The facts to which the laws of logic correspond] are to be compared with the facts that make measurement with a yardstick easy and useful." What's crucial here is that propositions having to do with conventions of measurement are not empirical, according to Wittgenstein:

> The role of propositions which deal with measures and are not 'empirical propositions'.—Someone tells me: "this stretch is two hundred and forty inches long". I say: "that's twenty foot, so it's roughly seven paces" and now I have got an idea of the length.—The transformation is founded on arithmetical propositions and on the proposition that 12 inches = 1 foot. (RFM VII, §355)

Conventions of measurement, like logical and mathematical propositions, depend on certain empirical conditions for their usefulness, but Wittgenstein makes it clear that he is not thinking of

conventions of measurement, e.g., '12 inches = 1 foot', as *stating* the facts that make these conventions useful, or possible.

> Does this mean that I have to say that the proposition '12 inches = 1 foot' asserts all those things which give measuring its present point? No. The proposition *is grounded in* a technique. And, if you like, *possible*. But it doesn't follow that its sense is to express these conditions. The opposite of that proposition, 'twelve inches = one foot' does not say that rulers are not rigid enough or that we don't all count and calculate in the same way. (RFM VII, §1)

It is clear then that Wittgenstein is, at least in part, comparing logic and mathematics to yardsticks because (1) like yardsticks, logic and mathematics importantly depend on empirical conditions for their usefulness, but (2) like '12 inches = 1 foot', propositions of logic and mathematics do not make claims about the general empirical conditions that make their use practical. Their sense is distinctive, and should not be identified with the sense of propositions which do state such empirical conditions directly, e.g., that yardsticks are natural and user-friendly for ordinary humans, that the material from which we construct yardsticks does not vary wildly depending on the temperature of its current environment, and so forth.

These texts are suggestive. But one might wonder whether the upshot should really be that Wittgenstein is attempting to drag us away from a view according to which logical and mathematical propositions make claims about the world in the way that empirical claims do. I think that the case is settled when we turn to the very texts that Maddy makes use of to support her interpretation of Wittgenstein's remarks on logic, in particular RFM I, §5. But it will be useful to compare the texts Maddy cites with Wittgenstein's immediately preceding remark about counting.

> Counting (and that means: counting like *this*) is a technique that is employed daily in the most various operations of our lives. And that is why we learn to count as we do: with endless practice, with merciless exactitude; that is why it is inexorably insisted that we shall all say "two" after "one", "three" after "two" and so on.—"But is this counting only a *use*, then; isn't there also some truth corresponding to this sequence?" (RFM I, §4)

Here Maddy would like to answer that, yes, there is a truth corresponding to the correctness of this sequence, namely, the contingent KF-structure of the world. But Wittgenstein retorts:

> The *truth* is that counting has proved to pay.[30] (RFM I, §4)

Again, this is meant to weaken the temptation to say that mathematics must correspond, represent, or report a distinctively mathematical reality, or aspect thereof. The only reality to which our counting practices correspond, Wittgenstein recommends, is the fact that these practices have proven useful. Various empirical conditions might in turn explain why these practices are indeed useful, but, like the measurement conventions associated with our yardstick, mathematical propositions do not make claims about these empirical conditions. This passage is followed by a related remark about logic.

> "But isn't there a truth corresponding to logical inference? Isn't it *true* that this follows from that?"—The proposition: "It is true that this follows from that" means simply: this follows from that. (RFM I, §5)

As we noted, and for the reasons mentioned earlier, Wittgenstein is wary of 'truth'. That is, in saying that "it's *true* that this follows from that", we are tempted to think that this is somehow fundamentally similar to the claim that "it's *true* that it will rain tomorrow".[31] Wittgenstein's

[30] Compare also: "You might say, "13 comes in one series, 15 in another." But isn't this what makes it a different series?—In one technique one follows, in the other, the other follows. [There would be] no discovery that 15 follows 12; it would just be a technique. Inventing a technique: 12, 14, 13, … This would be immensely impractical, inconvenient—but not wrong. … There is no discovery that 13 follows 12. That's our technique—we *fix*, we teach, our technique that way. If there is a discovery—it is that this is a valuable thing to do" (LFM, 83).

[31] Compare: "Why do you always want to look at mathematics under the aspect of finding and not of doing? It must have a great influence, that we use the words "right" and "true" and "wrong" and the form of statement, in calculating. (Head-shaking and nodding.)" (RFM VII, §5). That is, a potential source of our inclination to think of mathematics and logic as a kind of discovery, whereby we investigate the mathematical and logical features of reality, is the surface similarity of mathematical and empirical claims: that they are both called 'true', 'false', 'right', 'wrong', that they are the objects of belief, viz., that we say of both kinds of propositions that we 'believe' them, and so forth.

immediate reaction to this natural temptation is thus to say that "it's *true* that this follow from that" means simply "this follows from that". By removing the distracting element, we can focus on the proposition itself, and examine its role. The passage continues:

> And how do we use this proposition?—What would happen if we made a different inference—*how* should we get into conflict with truth? (RFM I, §5)

Maddy suggests that Wittgenstein opens up this question in a way that makes it amenable to Second Philosophical investigation. That is, different inferential practices would conflict with our interests in knowing and understanding the world, and they would conflict with very general features of reality, such as the fact that it has objective KF-structure. But this is to ignore the significance of the comparison that immediately follows the above passage.

> How should we get into conflict with truth, if our footrules were made of very soft rubber instead of wood and steel?—"Well, we shouldn't get to know the correct measurement of the table."—You mean: we should not get, or could not be sure of getting, *that* measurement which we get with our rigid rulers. … "But surely that isn't measuring at all!"—It is similar to our measuring and capable, in certain circumstances, of fulfilling 'practical purposes'. (A shopkeeper might use it to treat different customers differently.) (RFM I, 5)

The comparison is designed to draw our attention to the fact that, according to our logical conventions, surely "it's green" follows from "either it's red or it's green" and "it's not red", but it would be a mistake to think that the convention itself depends on anything beyond our practical interests, and the usefulness of the rule. The results from folks who use a rubber ruler might conflict with *our* measuring results, but their practice could be just as well grounded in their interests as our measuring practices are in our interests. The point, again, is to weaken the temptation to think that our logical conventions must report something about the world, beyond our practical requirements, beyond what is natural to us, and beyond our conventions. That is, to weaken the temptation to invoke objective KF-structure, let alone anything more specific.

It will help to examine, in a bit more detail, a case where Wittgenstein draws out the differences between empirical and logical or mathematical propositions. The insights we draw from this example can be used as a template for the numerous other cases that Wittgenstein discusses in Part I of the RFM.[32]

In §21, Wittgenstein reacts to a picture according to which logical implications are "already there" prior to one's thoughts about them, one simply has to make the relevant inferences. This is closely related to the Second Philosopher's picture of logical truth: Using Maddy's example from earlier, "it's green" already follows, indeed, *must* follow, from "it's either red or green" and "it's not red", prior to my recognizing this—it follows *objectively*. When I infer that "it's green", I come to a conclusion that is made true by the structure of the world and the truth of the premises.

> In his fundamental law Russell seems to be saying of a proposition: "It already follows—all I still have to do is, to infer it". Thus Frege somewhere says that the straight line which connects any two points is really already there before we draw it; and it is the same when we say that the transitions, say in the series + 2, have really already been made before we make them orally or in writing—as it were tracing them. (RFM I, §21)

Wittgenstein raises the further temptation to think that, by making the relevant inference, we "ascertain that such-and-such follows"—that we've grasped something that was, in some sense, "already there".

> "But we surely infer this proposition from that because it actually follows! We ascertain that it follows."—We ascertain that what is written here follows from what is written there. And this proposition is being used *temporally*. (RFM I, §23)

Wittgenstein's response is to replace a misleading description of what goes on in a proof with an ordinary, humdrum description of what happens: We see that what is written here follows from

[32] For instance, Wittgenstein's discussion of marbles (RFM I, §36), apples (RFM I, §37), tangrams (RFM I, §42f), a rectangle constructed out of parallelograms (RFM I, §50), the tangent of a visual curve (RFM I, §96), counting nuts (RFM I, §137), among others.

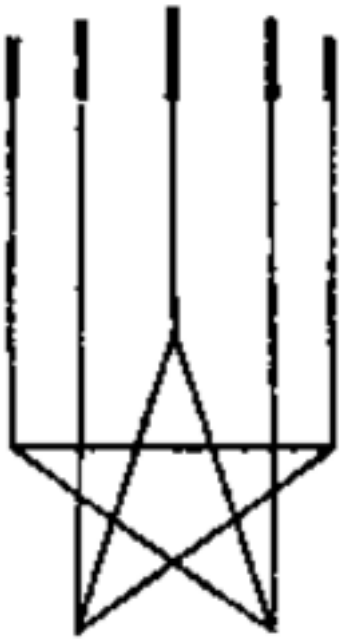
what is written there. This is a temporal proposition in that we are describing what in fact happens as I move through the proof: I look at one written sentence, pattern, picture, or whatever, *and then* move on to another one, recognizing that the latter things follow from the former, perhaps by saying "Aha! This follows", or simply by nodding my head. More importantly, ascertaining that "this follows", Wittgenstein insists, needs to be understood in terms of the role that the proof plays in the rest of one's activities. To get a better grip on this, we need a yet more specific case.

As an initial response to this sort of picture of logical inference—i.e., one according to which the conclusion *already follows*, I just have to ascertain that it does—Wittgenstein works through a particular example of a very simple mathematical proof. The proof is that a certain pattern of strokes, (a), is like-numbered with the pattern of angles on a pentagram, (b).

(a)

(b)

It is shown that the number of strokes is equal to the number of angles by drawing thin lines between the strokes in (a) and the angles in (b), illustrating a one-one correspondence in the pattern (c).



(c)

Wittgenstein asks: "Now what do I ascertain when I look at this figure? What I see is a star with threadlike appendages" (RFM I, §25). In order to understand that this is not merely a picture of "a star with thread-like appendages", that it is a mathematical *proof*, we need to examine the role that this picture plays in our language. As we'll see, the role of the pattern in (c) is carefully distinguished from the role of (c)-treated-as-an-empirical-demonstration.

Wittgenstein immediately offers a particular use for the pattern in (c).

> I can make use of the figure like this: five people stand arranged in a pentagon; against the wall are wands, like the strokes in (a); I look at the figure (c) and say: "I can give each of the people a wand". (RFM I, §26)

That is, now that we've accepted the pattern in (c) as a demonstration that (a) has the same number of lines as (b) has angles, we can apply this pattern to determine the sameness in number of other things. In the application that Wittgenstein has in mind, it can be used to determine that one can distribute a collection of wands equally to people standing at the corners of a pentagram. Wittgenstein seems to think that this sort of application partly reveals the distinctively mathematical role played by (c). This is amplified by contrasting (c) and its application with another case. One is presented with the arbitrary polygon, (d), and asked whether it has the same number of angles as the pattern (e) has strokes.

(d)

(e) IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII

Wittgenstein writes

> I can find out by correlating them whether I have as many angles in the top figure as strokes in the bottom one. (I do not know how it would turn out.) And so I can also say that by drawing projection-lines I have ascertained that there are as many strokes at the top of figure (c) as the star beneath has points. (Temporally!) In this way of taking it the figure is not like a mathematical proof (any more than it is a mathematical proof when I divide a bag of apples among a group of people and find that each can have just *one* apple). (RFM I, §27)

In other words, we could understand the process of drawing up (c) as a kind of experiment to see whether there are as many strokes in (a) as there are angles in (b), just as we could draw thin lines connecting the angles in the arbitrary polygon (d) with the arbitrary set of strokes (e). But this would make (c) a kind of "empirical" demonstration, rather than a mathematical proof. Why? Because I'm merely ascertaining something about *these lines* and *these angles*, as they're drawn on the page.

> I can however conceive figure (c) as a mathematical proof. Let us give names to the shapes of the patterns (a) and (b): let (a) be called a "hand", H, and (b) a "pentacle", P. I have proved that H has as many strokes as P has angles. And this proposition is once more non-temporal. (RFM I, §27)

Unlike ascertaining that the arbitrary polygon in (d) has the same number of angles as (e) has strokes, one ascertains something "timeless" with the demonstration illustrated by (c), and this is made clear when we call (a) a "hand", H, and (b) a "pentacle", P: We've shown, very generally, and timelessly, that H has as many strokes as P has angles. We could do the same with (d) and (e), but since these figures do not make for any immediately natural, easy, or obvious application, there is little motivation to do so. Until it has a natural application like (c), a "demonstration" that (d) has as many angles as (e) has strokes does not ascend to the status of *mathematical* demonstration. This allows (c) to play a distinctive role that was unavailable to (c)-understood-empirically.

> The proposition proved by (c) now serves as a new prescription for ascertaining numerical equality: if one set of objects has been arranged in the form of a hand and another as the angles of a pentacle, we say the two sets are equal in number. (RFM I, §30)

The timeless, and completely general, proposition that H has as many strokes as P has angles, gains its mathematical status in part because it can be used as a prescription for ascertaining numerical equality. The empirical version of (c) had a merely descriptive status, telling us about particular

figures on a page, whereas (c), understood mathematically, ascends to the status of a rule. That is, since it is *used* as a rule for ascertaining numerical equality, (c) is mathematical, rather than empirical.

With (c), one does not "merely… correlate H and P and see that they are the same in number":

> if they were so in one case, how do I know that they will be so again now?—"Why, because it is of the *essence* of H and P to be the same in number."—But how can you have brought *that* out by correlating them? (I thought the counting or correlation merely yielded the result that these two groups before me were—or were not—the same in number.) (RFM I, §31)

Wittgenstein here addresses the temptation to think that one engages in a kind of experiment when drawing up (c), the result of which is that there are as many strokes in (a) as there are angles in (b). But how then, from this single experiment, are we able to apply the figure so generally to determine the numerical equality of other collections? The response is that H and P are, by their very *essence*, the same in number (I'll explain in a moment how we should interpret Wittgenstein's use of 'essence' here). Wittgenstein responds that this cannot be shown simply by correlating the strokes with the angles in this particular case. Just as one does not show that it is part of the *essence* of a bag of apples that it can be divided evenly among four people by, at a particular time, correlating the apples in the bag with people in a particular group, one cannot show that H and P are equal in number simply by correlating this particular shape with this particular collection of strokes. Wittgenstein's interlocutor is right that (c) gains its mathematical status because of the essences of H and P, but wrong that this is discovered as a matter of experiment, i.e., as a matter of correlating these particular strokes and these particular angles.

> I might also say as a result of the proof: "From now on an H and a P are called 'the same in number'".

> Or: The proof doesn't *explore* the essence of the two figures, but it does express what I am going to count as belonging to the essence of the figures from now on.—I deposit what belongs to the essence among the paradigms of language.
>
> The mathematician creates *essences.* (RFM I, §32)

Unlike the empirical claim that there are as many strokes in (a) as there are angles in (b), the mathematical version of (c) reveals that, as a matter of the essence of H and P, there are as many strokes in H as angles in P. But, the essences of H and P are *created* rather than "explored" or "discovered" by the mathematician, and used as a prescription, a technique, and a method for ascertaining other numerical equalities.

To sum up: the mathematical version of (c) is distinctively mathematical in that it is a-temporal, prescriptive, and is not accepted merely as a matter of correlating *these* strokes with *these* angles—it involves the creation of essences on the part of the mathematician. We need not impose a "metaphysical" reading on Wittgenstein's use of 'essence' here. H and P are 'essentially' the same in number in that (i) we *treat them* as essentially being the same number, (ii) we *treat* two things that are not the same in number as automatically not H and P, and (iii) we thereby use (c) as a schema to ascertain the numerical equality of other things, e.g., wands and people standing at the points of a pentagram. In the immediately following remark, we come full circle, back to Wittgenstein's initial concerns about a picture of logical inference, according to which something follows before we have anything to say about it.

> When I say "This proposition follows from that one", that is to accept a rule. The acceptance is *based* on the proof. That is to say, I find this chain (this figure) acceptable as a *proof.* (RFM I, §33)

We "ascertain" that something follows, only in the temporal sense that, as a matter of fact, we work through a proof by looking at some bit or writing, and *then* look at another bit of writing; as

a result, we treat the latter as following from the former. Treating the conclusion as *following from* the antecedent claims is, in turn, exemplified by my acceptance of a rule, a technique, and a method. For this reason, though perhaps others,[33] the acceptance of a logical inference, or a logical proposition, is fundamentally different from my acceptance of the empirical claim that '*this* pattern has as many strokes as *this* pattern has angles', 'it's going to rain tomorrow', or 'there are Bengal tigers in Nepal'.

[33] As the RFM involves a huge number of cases, in many of which Wittgenstein carefully distinguishes mathematical or logical and empirical propositions, the case-study here should not be seen as exhaustively drawing out the distinguishing features of logic and mathematics. See fn. 28.

VI. CONCLUSION

Maddy cites the following passage in support of her interpretation of Wittgenstein on logic. In light of my arguments, I think it should be clear that her interpretation is questionable.

> But still, I must only infer what really *follows*!—Is this supposed to mean: only what follows, going by the rules of inference; or is it supposed to mean: only what follows, going by *such* rules of inference as somehow agree with some (sort of reality)? Here what is before our minds in a vague way is that this reality is something very abstract, very general, and very rigid. Logic is a kind of ultra-physics, the description of the 'logical structure' of the world, which we perceive through a kind of ultra-experience. (RFM I, §8)

Maddy claims,

> I hope it's…clear that despite the Second Philosopher's talk of 'the "logical structure" of the world'—though she would omit the scare quotes—it would take a considerable stretch to locate her position in this quotation. 'Very general' fits well, but 'very abstract' less so, and 'very rigid' is hard to interpret at all along second-philosophical lines. 'Ultra-physics' sounds stronger than merely more general than physics, and 'ultra-experience' or 'perceiv[ing] with the understanding' would be an odd way of describing the impression of obviousness that comes with the Second Philosopher's simple logical truths. In fact, the quotation seems intended to express a yearning for something more than the Second Philosopher's ordinary truths about the world. (Maddy 2014, 82, note 1)

Maddy is quite right that the picture articulated by Wittgenstein in RFM I, §8 is not *exactly* the picture that the Second Philosopher has in mind when she talks of logical structure. However, I think we can see that Wittgenstein would have been equally concerned with the Second Philosopher's temptations when we observe the more important theme that runs through Part I of the Remarks: To study the use, function, and application of mathematical and logical propositions so that one can properly differentiate them from empirical claims. The function of these remarks

is in part to demystify logic, and weaken the temptation to engage in any kind of theorizing about the reality to which it allegedly corresponds. It doesn't matter whether that reality is thought to be 'abstract', 'necessary', 'a priori', or whatever; Wittgenstein is reacting to any picture of a distinctively logical reality, and undermining it by reacting to its source, namely, a misleading analogy between empirical and logical propositions, empirical and logical truths. Again, I think that Wittgenstein's reported remarks in the LFM stabilize this point.

> Consider Professor Hardy's article ("Mathematical Proof") and his remark that "to mathematical propositions there corresponds—in some sense, however sophisticated—a reality". (The fact that he said it does not matter; what is important is that it is a thing which lots of people would like to say.) Taken literally, this seems to mean nothing at all—*what* reality? I don't know what this means.—But it is obvious what Hardy compares mathematical propositions with: namely physics. (LFM, 239)

What I suggest is that we read RFM I, §8 as warning against a picture according to which mathematical propositions "correspond to—in some sense, however sophisticated—a reality". But then the remark applies to the Second Philosophical perspective just as well as any other metaphysical proposal about the ground of logical truth.[34] And, as we have seen, this is not a mere conviction of Wittgenstein's, one without philosophical significance that we can ignore as a mere

[34] Maddy might resist this argument, claiming that Hardy is interested in a 'spooky' or 'mysterious' kind of reality, whereas her KF-structure is simply "ordinary reality" under a very general description. Hence, Wittgenstein could very well be reacting to a picture of mathematical reality that does not in any way resemble the Second Philosopher's theory. Although Hardy claimed that mathematical truth is "immutable and unconditional", which the Second Philosopher explicitly rejects, Hardy's own statements about what mathematical reality might consist in are remarkably weak: "In *some* sense, mathematical truth is part of objective reality. …[Mathematical propositions] are not convenient working hypotheses, or half-truths about the Absolute, or collections of marks on paper, or classes of noises summarizing reactions of laryngeal glands. They are, in one sense or another, however elusive and sophisticated that sense may be, theorems concerning reality" (Hardy 1929, p. 4). Hardy is not claiming that mathematical reality *is* elusive, nor that it *is* (somehow) 'sophisticated'. He's simply making the point that, whatever the full metaphysical story turns out to be, mathematical propositions say something about objective reality. And it is this very simple picture of mathematics that Wittgenstein seems to be reacting to. If the Second Philosopher's "ordinary reality" is somehow less spooky or mysterious than what Hardy has in mind, then she'll need to explain why, which will be difficult, as Hardy's suggestion is already quite underspecified.

fact of biography. If my interpretation is correct, Wittgenstein seeks to undermine this sort of picture by reacting against the analogy that gives rise to it: namely, a misleading analogy between empirical and mathematical propositions. Of course, what we have said thus far does not itself constitute the most comprehensive and scrupulous version of such a therapy. But I hope that I have given ample reason to think that the resources for such a therapy are to be found in the RFM, and that it applies directly to the Second Philosopher's inquiry into the ground of logical truth. Hence, Wittgenstein's anti-scientism aside, there are substantive philosophical grounds for Wittgenstein's departure from the Second Philosophical view of logic.

I also hope it will be clear that comparing Wittgenstein's perspective to that of the Second Philosopher served a more positive task, namely, that of understanding Wittgenstein's own views about mathematics and logic. As Wittgenstein famously wrote "Philosophy is not a body of doctrine but an activity" (TLP, 4.112). The activity of the early Wittgenstein was the clarification of thoughts by examining the general form of propositions, and the logical structure of the world. The activity of the later Wittgenstein is therapeutic: To investigate and understand the sources of philosophical confusion so that we might put it aside. Examining how Wittgenstein would have reacted to the Second Philosopher's inquiry—how he, qua therapist, would diagnose the source of her, qua patient's, perplexity and philosophical confusion—thus allows for greater understanding of Wittgenstein's distinctive philosophical methods, as well as his unique perspective on logic and mathematics.

My arguments can thus be seen as making a more general point about a potential source of confusion: A hasty and misleading analogy between science, and logic or mathematics. The Second Philosopher is explicit that her inquiry relies on such an analogy, but I doubt that she is alone. Wittgenstein's remarks on logic and mathematics bear on any philosopher's temptation to

investigate mathematical and logical reality, especially if one's temptation crucially relies on the superficial similarities of propositions like (1) – (3) introduced at the beginning of this paper.

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